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A New Approach to Find Minimal Dominating Set of an Interval Graph

Arnab Kumar Das [1], Dr Bichitra Kalita (RTD) [2]

Asst Professor
Dept of Computer Application
GIMT, Guwahati-17
arnab_kumar@gimt-guwahati.ac.in

Associate Professor and Head
Dept. of Computer Application
Assam Engineering College, Jalukbari
bichitra1_kalita@rediffmail.com

Abstract: The problem of finding a minimal dominating set or externally stable set in a graph is a classical NP complete problem in computational complexity theory. Till now there is no efficient algorithm that finds a smallest dominating set for a given graph. Several greedy algorithms have been proposed to find the minimal dominating set on a special class of graph called interval graph. Interval graphs are very important in finding combinatorial structures and various applications have been found in different fields. In this paper the problem of computing minimum dominating sets of n intervals on the real lines have been studied. A new algorithm is proposed which will produce solutions of finding dominating set which are optimal up to a certain factor.

Key words: Interval graph, Dominating set, Minimal dominating set.

1. INTRODUCTION:

Interval graphs have been occupying the attention of many researchers for many years for their various applications in different fields, quite a few of them are resource allocation problem, traffic control, ecology, biology etc.
On the other hand, the dominating sets are one of the concepts in graph theory where some problem of optimizations, design and analysis of communication networks, and social sciences and military surveillance has been studied.
Various works have been found on interval graphs. Interval graphs are useful in modeling resource allocation problems in operations research. Minimum dominating set can be used to simplify connectivity management in wireless adhoc network which is used in many applications such as automated battlefield
and rescue and disaster relief. However to find minimum dominating set is a NP hard problem. Some approximation algorithm for it had been proposed [1][2] but with a poor approximation ratio with high time and message complexity. But according to the authors' research it is found that many linear time and polynomial time greedy algorithm[3] was proposed to find the minimal dominating set for a special class of graph known as interval graph. Interval graph is a very important graph, and has a lot of advantages in some special network management. Some linear time algorithms for recognition of proper interval graph had been proposed [4][5][6]. The domination is one of the parameters in graphs which has a great importance in modern circuit designing systems. K Dhanalakshmi and B Maheswari[8] discuss matching domination number of interval graphs and propose an algorithm for finding matching dominating sets in interval graphs. Dr. A. Sudhakaraiah* V. Rama Latha E.Gnana Deepika[9] has proposed another algorithm for split restrained dominating set of an interval graph.They explored further in this area and propose another algorithm for finding split dominating set of an interval graph[10]. B. S. Panda and D. Pradhan[11] proposed a polynomial time reduction that proves the NP-completeness of the restrained domination problem for undirected path graphs, chordal bipartite graphs, circle graphs, and planar graphs. B. S. Panda and Arti Pandey [12] have studied the dominator chromatic number for the proper interval graphs and block graphs.
The present work is intended to discuss the problem of vertex domination in graphs. In this paper we investigate the minimum domination number of an interval graph. For this we propose an algorithm which will accept an interval graph as input and produces the minimal dominating set as output. This concept can be applied in situations like representing a shift of a worker, where a minimum dominating set will represents the smallest number of supervisor to be on duty so that each junior worker can consult one supervisor during his/her shift.

2. DEFINITIONS

2.1 Interval Graph

An interval graph is the intersection graph of a family of intervals on the real line. It has one vertex for each interval in the family, and an edge between every pair of vertices corresponding to intervals that intersect.

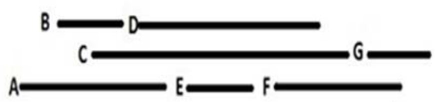

Fig 1

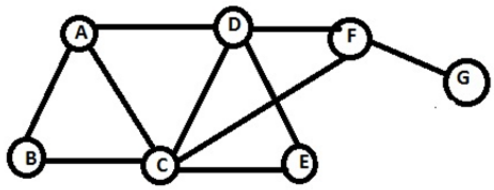

Fig: 2

Formally, an interval graph G is an undirected graph formed from a family of intervals Si, i = 0, 1, 2, ... by creating one vertex vi for each interval Si, and connecting two vertices vi and vj by an edge whenever the corresponding two sets have a nonempty intersection, that is, the edge set

$E(G) = \{\{vi, vj\} \mid Si \cap Sj \neq \emptyset\}$.In the above diagram, the Interval graph of Figure 2 is obtained from the intervals in the real line of Figure 1

From this construction one can verify a common property held by all interval graphs. That is, graph G is an interval graph if and only if the maximal cliques of G can be ordered M1, M2, ..., Mk such that for any $v \in M_i \cap M_k$, where $i < k$, it is also the case that $v \in Mj$ for any Mj, $i \leq j \leq k$

2.2 Dominating Set

A dominating set for a graph G = (V, E) is a subset D of V such that every vertex not in D is adjacent to at least one member of D. The domination number $\gamma(G)$ is the number of vertices in a smallest dominating set for G.

The dominating set problem concerns testing whether $\gamma(G) \leq K$ for a given graph G and input K; it is a classical NP-complete decision problem .

3. PROPOSED ALGORITHM FOR FINDING MINIMAL DOMINATING SET OF AN INTERVAL GRAPH.

Input: An Interval Graph

Output: A minimal Dominating set

Step 1: For all vertex Vi corresponding to interval Si in G, find I (Vi) where I (Vi) is the set of intervals that intersects Vi.

Step 2: Find the largest set of intervals. If more than one set is having the same number of intervals then arbitrarily select one. Say the arbitrarily selected set is I (V_k).

Step 3: Set P=max (V_k) where max (V_k) is the vertex corresponding to the largest interval in I (V_k).

Step 4: Set M=Next (P) where Next (P) is the vertex that corresponds to the next interval of P in the real line.
If Next (P) =NULL then M=P

Step 5: Perform V_k U M which is the resultant minimum dominating set.

4.ILLUSTRATION :

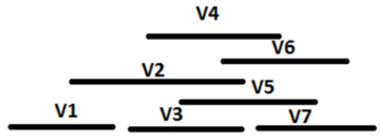

Fig:3

Step 1**:** For all vertices V_1 to V_7 in Fig 3

$I(V_1)= \{ V_1,V_2\}$

$I(V_2)=\{ V_1, V_2,V_3,V_4,V_5,V_6\}$

$I(V_3)=\{ V_2,V_3,V_4,V_5,V_6\}$

$I(V_4)=\{ V_2,V_3,V_4,V_5,V_6\}$

$I(V_5)=\{ V_2,V_3,V_4,V_5,V_6, V_7\}$

$I(V_6)=\{ V_2,V_3,V_4,V_5,V_6, V_7\}$

$I(V_7)=\{ V_5,V_6, V_7\}$

Step 2:Here $I(V_2)$, $I(V_5)$, $I(V_6)$ and $I(V_7)$ are having the largest number of intervals. Let us arbitrarily select $I(V_2)$.

Step 3: P = max $(V_2) = V_6$

Step 4: M= Next (P) =Next $(V_6) = V_7$.

Step 5: Thus V_k U M= V_2 U $V_7 = \{ V_2, V_7\}$

Thus in our example Minimal dominating set is $\{ V_2, V_7\}$

4.1Description and Explanation of the algorithm:

In this algorithm for each vertex corresponding to the interval we first find the set of intervals that intersect with it. Then we arbitrarily select the largest Interval set. Then we find the largest interval in the set. After getting the largest interval in the Interval set we find the next interval after it in the real line. Then we perform union of vertex corresponding to the largest interval set with vertex corresponding to the largest interval set which is the required minimum dominating set.

4.2 Implementation of the Proposed Algorithm

We implement the proposed algorithm in Visual Studio .Net using Visual C++. For this we create a windows Form application and design the form as shown in Fig 4.

We first have to enter how many intervals we want to create. Then for each interval we have to enter the min value and max value. The windows form to enter number of intervals and their min value and max value is shown Fig 5

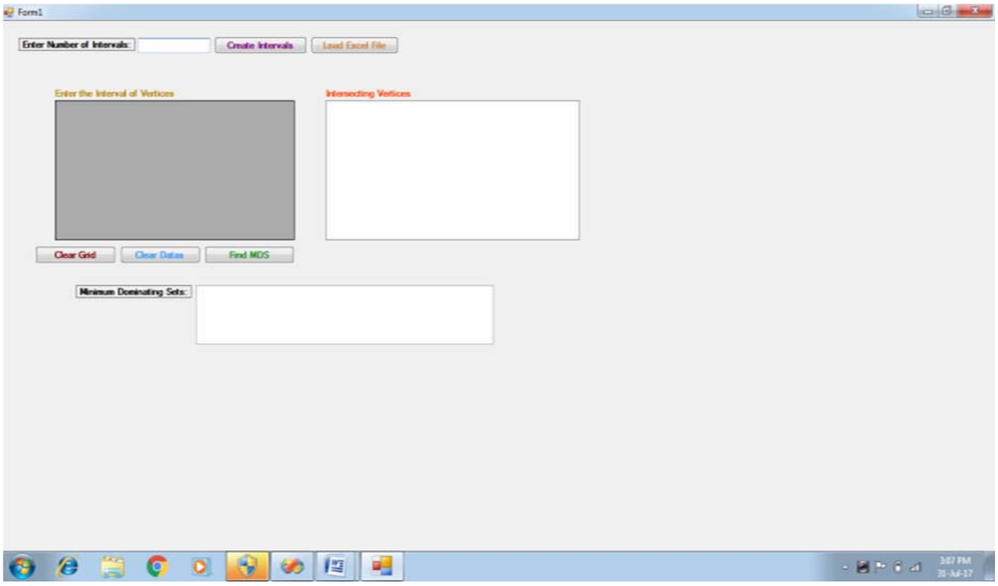

Fig4: Windows form to find Minimal dominating set

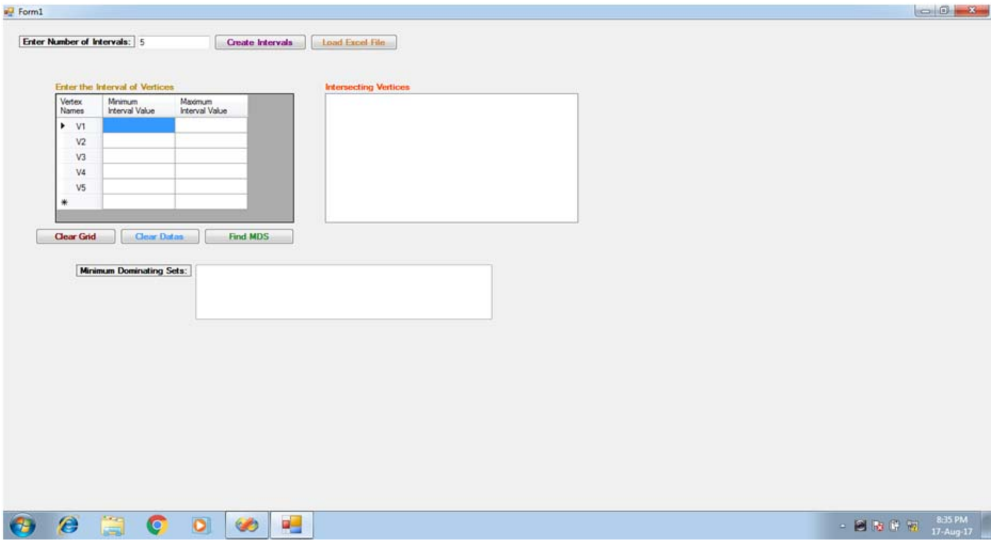

Fig 5: Form to enter the minimum and maximum value of Intervals

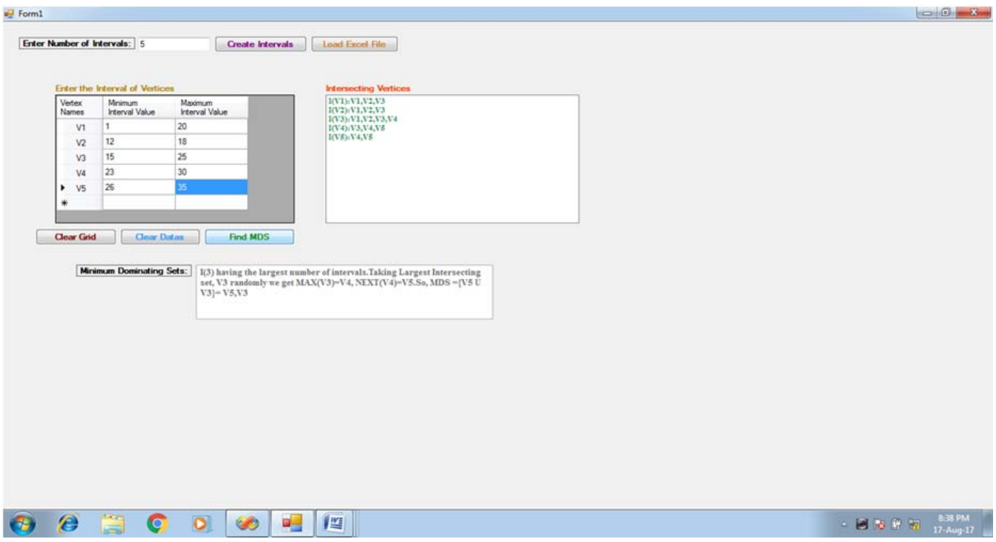

Fig 6: Minimal dominating set of the given intervals.

Based on these values we will find the intersecting intervals as follows.

For two intervals a and b having min value and max value as amin, amax, bmin and bmax the following condition is checked to find whether they are intersecting with each other or not.

```
((amin>=bmin && amin<=bmax) ||
(amax>=bmin && amax<=bmax) ||
(amin<=bmin && amax>=bmax) ||
(amin>=bmin && amax<=bmax)))
```

Once we find the intersecting set of vertices then we find the largest intersecting set. It may happen that we may get more than one largest intersecting set . In such situation we have to randomly select a set. This is achieved by using the "rand" function which generates a pseudorandom number. After randomly selecting the largest intersecting set we find the maximum of the vertex corresponding to the largest intersecting set. This is achieved by comparing the max value of the vertices in the intersecting set.After getting the max vertex we find the next of the maxvertex by finding the maxium value among the vertices present in the intersecting set corresponding to the max vertex. Finally the minimum dominating set is obtained by performing the union between the vertex corresponding to the largest intersecting set and vertex corresponding to the next of the maximim vertex set.The final outcome is shown below in Fig 6.

It is also to be mentioned that the program has the provision to load excel files(with or without header) also in case we need to enter a large number of intervals.

4.3 Time and space complexity analysis:

The time complexity and space complexity of the algorithm will be O (V^2). This can be explained as follows.

The algorithm first find the intersecting set based on the interval. So the vertex corresponding to an interval has to check with all the vertices corresponding to the other intervals whether it is intersecting with it or not. That is if there are n vertices then for a particular vertex V_i it needs to check with (n-1) vertices whether it is intersecting or not. Moreover the vertex V_i itself is included in the intersecting set of $V_{i.}$ Thus we need V*V i.e. V^2 comparisons. Also after finding the intersecting we need to find the largest intersecting set .Selection of the largest set will take time O(1). That is it will take constant time because irrespective of the vertex which is selected the time required for this operation will be same. Also the time required to find the vertex having the maximum value will take V *V comparisons. Because the max value of all vertices in the largest intersecting set need to be compared. Other V^2 comparisons will be required to find the next of the max vertex. Finally the time required to perform the union operation is O(V) i.e the total number of vertices in both the set. Thus the total time complexity of the algorithm is O (V^2) + O (1) = O (V^2).

4.4 Correctness of the algorithm:

The algorithm is correct in the sense that it produces a minimal dominating set of an interval graph in a very simple way. First it make sure that it finds all the neighbour of a vertex corresponding to an interval in the real line. Then it finds the vertex having the highest number of neighbour. Tie between vertex is broken by arbitrarily selecting a vertex. The largest interval corresponding to a vertex which is adjacent to the vertex having the highest number of neighbor can be easily find out by scanning the real line. Moreover the vertex corresponding to the next interval can also be find out by scanning the real line.

5. Conclusion

The minimum dominating set (MDS) problem is an important and well-studied combinatorial problem which is NP-hard for some classes of graphs. In this paper we have given an approximation algorithm which is able to find the minimum dominating set of an interval graph. There are also scope to use the minimum dominating set for routing in Mobile Adhoc Network (MANET) .In future we will try to focus on the use of MDS to perform power efficient routing in MANET.

References

[1] Bo Gao,Yuhang Yang,and Huiye Ma.A new distributed approximation algorithm for constructing minimum connected dominating set in wireless ad hoc networks[J]. International Journal of Communication, 2005,18(8):743-762.

[2] P Rajiv Gandhi, Srinivasan Parthasarathy. Distributed algorithms for connected domination in wireless networks[J]. Journal of Parallel and Distributed Computing, , 2007,67(7):848-862.

[3] Solving Minimum Connected Dominating Set on Proper Interval Graph Jinqin Tian, Hongsheng Ding Beifang University of Nationalities Yinchuan, China ,2013 Sixth International Symposium on Computational Intelligence and Design,IEEE

[4] Panda B. S., Das Sajal K. A linear time recognition algorithm for proper interval graphs[J]. Information Processing Letters,2003,87(3):153-161.

[5] Jorgen Bang-Jensen, PJing Huang.Recognizing and representing proper interval graphs in parallel using merging and sorting[J]. Discrete Applied Mathematics,2007,155(4):442-456.

[6] Sheng-Lung Peng, Chi-Kang Chen.On the interval completion of chordal graphs[J]. Discrete Applied Mathematics,2006,154(6):1003-1010.

[7] M. Pal, S. Mondal, D. Bera, and T. K. Pal, "An optimal parallel algorithm for computing cut vertices and blocks on interval graphs," *International Journal of Computer Mathematics*, vol. 75, no. 1, pp. 59–70, 2000.

[8] K.Dhanalakshmi, B.Maheswari, Matching Dominating Sets of Interval Graphs, International Journal of Computer Applications (0975 – 8887) Volume 88 – No.5, February 2014

[9] A.Sudhakaraiah, E.GnanaDeepika, V.Ramalatha : Split Restrained Dominating Set of an Interval Graph using an Algorithm, International Journal of Applied Information Systems (IJAIS) – ISSN : 2249-0868 Foundation of Computer Science FCS, New York, USA Volume 4– No.9, December 2012 .

[10] Dr. A. Sudhakaraiah,V. Rama LathaE.GnanaDeepika: Split Dominating Set of an Interval Graph Using an Algorithm., International Journal Of Scientific & Engineering Research, Volume 3, Issue 6, June-2012 1 ISSN 2229-5518

[11] B. S. Panda and D. Pradhan: A linear time algorithm to compute a minimum restrained dominating set in proper interval graphs, Discrete Mathematics, Algorithms and Applications Vol. 7, No. 2 (**2015**) 1550020 (21pages)cWorld Scientific Publishing Company.

[12] B. S. Panda and Arti Pandey: On the dominator coloring in proper interval graphs and block graphs, Discrete Mathematics, Algorithms and Applications